SUBJECT CONTENT

Knowledge of the content of O-Level Mathematics syllabus is assumed in the syllabus below and will not be tested directly, but it may be required indirectly in response to questions on other topics.

Topic/Sub-topics		Content
ALGEBRA		
A1	Equations and inequalities	• Conditions for a quadratic equation to have: (i) two real roots (ii) two equal roots (iii) no real roots and related conditions for a given line to: (i) intersect a given curve (ii) be a tangent to a given curve (iii) not intersect a given curve • Conditions for $ax^2 + bx + c$ to be always positive (or always negative) • Solving simultaneous equations in two variables with at least one linear equation, by substitution • Relationships between the roots and coefficients of a quadratic equation • Solving quadratic inequalities, and representing the solution on the number line
A2	Indices and surds	• Four operations on indices and surds, including rationalising the denominator • Solving equations involving indices and surds
A3	Polynomials and Partial Fractions	• Multiplication and division of polynomials • Use of remainder and factor theorems • Factorisation of polynomials • Use of: – $a^3 + b^3 = (a + b)(a^2 – ab + b^2)$ – $a^3 – b^3 = (a – b)(a^2 + ab + b^2)$ • Solving cubic equations • Partial fractions with cases where the denominator is no more complicated than: – $(ax + b)(cx + d)$ – $(ax + b)(cx + d)^2$ – $(ax + b)(x^2 + c^2)$
A4	Binomial expansions	• Use of the Binomial Theorem for positive integer n • Use of the notations $n!$ and $\binom{n}{r}$ • Use of the general term $\binom{n}{r}a^{n-r}b^r$, $0 < r \leqslant n$ (knowledge of the greatest term and properties of the coefficients is not required)

Topic/Sub-topics		Content
A5	Power, Exponential, Logarithmic, and Modulus functions	• Power functions $y = ax^n$ where n is a simple rational number, and their graphs • Exponential and logarithmic functions a^x, e^x, $\log_a x$, $\ln x$ and their graphs, including: – laws of logarithms – equivalence of $y = a^x$ and $x = \log_a y$ – change of base of logarithms • Modulus functions $\|x\|$ and $\|f(x)\|$ where $f(x)$ is linear, quadratic or trigonometric, and their graphs • Solving simple equations involving exponential, logarithmic and modulus functions
GEOMETRY AND TRIGONOMETRY		
G1	Trigonometric functions, identities and equations	• Six trigonometric functions for angles of any magnitude (in degrees or radians) • Principal values of $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ • Exact values of the trigonometric functions for special angles $(30°, 45°, 60°)$ or $\left(\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}\right)$ • Amplitude, periodicity and symmetries related to the sine and cosine functions • Graphs of $y = a\sin(bx) + c$, $y = a\sin\left(\frac{x}{b}\right) + c$, $y = a\cos(bx) + c$, $y = a\cos\left(\frac{x}{b}\right) + c$ and $y = a\tan(bx)$, where a is real, b is a positive integer and c is an integer. • Use of the following – $\frac{\sin A}{\cos A} = \tan A$, $\frac{\cos A}{\sin A} = \cot A$, $\sin^2 A + \cos^2 A = 1$, $\sec^2 A = 1 + \tan^2 A$, $\operatorname{cosec}^2 A = 1 + \cot^2 A$ – the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ – the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$ – the expression for $a\cos\theta + b\sin\theta$ in the form $R\cos(\theta \pm \alpha)$ or $R\sin(\theta \pm \alpha)$ • Simplification of trigonometric expressions • Solution of simple trigonometric equations in a given interval (excluding general solution) • Proofs of simple trigonometric identities

Topic/Sub-topics		Content
G2	Coordinate geometry in two dimensions	• Condition for two lines to be parallel or perpendicular • Midpoint of line segment • Area of rectilinear figure • Graphs of parabolas with equations in the form $y^2 = kx$ • Coordinate geometry of circles in the form: – $(x-a)^2 + (y-b)^2 = r^2$ – $x^2 + y^2 + 2gx + 2fy + c = 0$ (excluding problems involving 2 circles) • Transformation of given relationships, including $y = ax^n$ and $y = kb^x$, to linear form to determine the unknown constants from a straight line graph
G3	Proofs in plane geometry	• Use of: – properties of parallel lines cut by a transversal, perpendicular and angle bisectors, triangles, special quadrilaterals and circles♦ – congruent and similar triangles♦ – midpoint theorem – tangent-chord theorem (alternate segment theorem)
Calculus		
C1	Differentiation and integration	• Derivative of f(x) as the gradient of the tangent to the graph of y = f(x) at a point • Derivative as rate of change • Use of standard notations $\mathrm{f}'(x), \mathrm{f}''(x), \frac{\mathrm{d}y}{\mathrm{d}x}, \frac{\mathrm{d}^2y}{\mathrm{d}x^2}\left[=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\right]$ • Derivatives of x^n, for any rational n, sin x, cos x, tan x, e^x, and ln x, together with constant multiples, sums and differences • Derivatives of products and quotients of functions • Derivatives of composite functions • Increasing and decreasing functions • Stationary points (maximum and minimum turning points and stationary points of inflexion) • Use of second derivative test to discriminate between maxima and minima • Applying differentiation to gradients, tangents and normals, connected rates of change and maxima and minima problems • Integration as the reverse of differentiation • Integration of x^n, for any rational n, sin x, cos x, $\sec^2 x$ and e^x, together with constant multiples, sums and differences • Integration of $(ax + b)^n$, for any rational n, $\sin(ax + b)$, $\cos(ax + b)$, and e^{ax+b} • Definite integral as area under a curve • Evaluation of definite integrals • Finding the area of a region bounded by a curve and line(s) (excluding area of region between two curves) • Finding areas of regions below the x-axis • Application of differentiation and integration to problems involving displacement, velocity and acceleration of a particle moving in a straight line

♦ These are properties learnt in O Level Mathematics.

MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

MATHEMATICAL NOTATION

The list which follows summarises the notation used in Cambridge's Mathematics examinations. Although primarily directed towards A-Level, the list also applies, where relevant, to examinations at all other levels.

1. *Set Notation*

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
$\{x: \ldots\}$	the set of all x such that
$\mathrm{n}(A)$	the number of elements in set A
$\varnothing$	the empty set
$\mathscr{E}$	universal set
A'	the complement of the set A
$\mathbb{Z}$	the set of integers, {0, ±1, ±2, ±3, …}
$\mathbb{Z}^+$	the set of positive integers, {1, 2, 3, …}
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$
$\mathbb{Q}_0^+$	the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \geqslant 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R}: x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R}: x \geqslant 0\}$
$\mathbb{R}^n$	the real n tuples
$\mathbb{C}$	the set of complex numbers
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\not\subseteq$	is not a subset of
$\not\subset$	is not a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leqslant x \leqslant b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leqslant x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leqslant b\}$
(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$

2. *Miscellaneous Symbols*

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\propto$	is proportional to
$<$	is less than
$\leqslant$; $\not>$	is less than or equal to; is not greater than
$>$	is greater than
$\geqslant$; $\not<$	is greater than or equal to; is not less than
∞	infinity

3. *Operations*

$a + b$	a plus b
$a - b$	a minus b
$a \times b, ab, a.b$	a multiplied by b
$a \div b, \frac{a}{b}, a/b$	a divided by b
$a : b$	the ratio of a to b
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + ... + a_n$
$\sqrt{a}$	the positive square root of the real number a
$\lvert a \rvert$	the modulus of the real number a
$n!$	n factorial for $n \in \mathbb{Z}^+ \cup \{0\}$, $(0! = 1)$
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$, for $n, r \in \mathbb{Z}^+ \cup \{0\}$, $0 \leqslant r \leqslant n$
	$\frac{n(n-1)...(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{Z}^+ \cup \{0\}$

4. *Functions*

f	function f
$\mathrm{f}(x)$	the value of the function f at x
$\mathrm{f}: A \to B$	f is a function under which each element of set A has an image in set B
$\mathrm{f}: x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
$\mathrm{g} \circ \mathrm{f}$, gf	the composite function of f and g which is defined by $(\mathrm{g} \circ \mathrm{f})(x)$ or $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
$\lim_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
Δx ; δx	an increment of x
$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
$\mathrm{f}'(x)$, $\mathrm{f}''(x)$, …, $\mathrm{f}^{(n)}(x)$	the first, second, … nth derivatives of $\mathrm{f}(x)$ with respect to x
$\int y \mathrm{d}x$	indefinite integral of y with respect to x
$\int_a^b y \mathrm{d}x$	the definite integral of y with respect to x for values of x between a and b
$\dot{x}, \ddot{x}, \ldots$	the first, second, …derivatives of x with respect to time

5. *Exponential and Logarithmic Functions*

e	base of natural logarithms
e^x, $\exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$	natural logarithm of x
$\lg x$	logarithm of x to base 10

6. *Circular Functions and Relations*

$\sin$, $\cos$, $\tan$, cosec, $\sec$, $\cot$	the circular functions
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ cosec^{-1}, $\sec^{-1}$, $\cot^{-1}$	the inverse circular functions

7. *Complex Numbers*

i	square root of –1
z	a complex number, $z = x + \mathrm{i}y$ $= r(\cos\theta + \mathrm{i}\sin\theta),\ r \in \mathbb{R}_0^+$ $= r\mathrm{e}^{\mathrm{i}\theta},\ r \in \mathbb{R}_0^+$
$\mathrm{Re}\, z$	the real part of z, $\mathrm{Re}\,(x + \mathrm{i}y) = x$
$\mathrm{Im}\, z$	the imaginary part of z, $\mathrm{Im}\,(x + \mathrm{i}y) = y$
$\lvert z \rvert$	the modulus of z, $\lvert x + \mathrm{i}y \rvert = \sqrt{x^2 + y^2}$, $\lvert r(\cos\theta + \mathrm{i}\sin\theta) \rvert = r$
$\arg z$	the argument of z, $\arg(r(\cos\theta + \mathrm{i}\sin\theta)) = \theta$, $-\pi < \theta \leqslant \pi$
z^*	the complex conjugate of z, $(x + \mathrm{i}y)^* = x - \mathrm{i}y$

8. *Matrices*

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the square matrix $\mathbf{M}$
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
$\det \mathbf{M}$	the determinant of the square matrix $\mathbf{M}$

9. *Vectors*

$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of the vector $\mathbf{a}$
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$\lvert \mathbf{a} \rvert$	the magnitude of $\mathbf{a}$
$\lvert \overrightarrow{AB} \rvert$	the magnitude of $\overrightarrow{AB}$
$\mathbf{a}.\mathbf{b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a} \times \mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

10. *Probability and Statistics*

A, B, C, etc.	events
$A \cup B$	union of events A and B
$A \cap B$	intersection of the events A and B
$\mathrm{P}(A)$	probability of the event A
A'	complement of the event A, the event 'not A'
$\mathrm{P}(A \mid B)$	probability of the event A given the event B
X, Y, R, etc.	random variables
x, y, r, etc.	value of the random variables X, Y, R, etc.
$x_1,\ x_2, \ldots$	observations
$f_1,\ f_2, \ldots$	frequencies with which the observations, x_1, x_2 …occur
$\mathrm{p}(x)$	the value of the probability function $\mathrm{P}(X = x)$ of the discrete random variable X
$p_1,\ p_2 \ldots$	probabilities of the values $x_1,\ x_2, \ldots$of the discrete random variable X
$\mathrm{f}(x)$, $\mathrm{g}(x)\ldots$	the value of the probability density function of the continuous random variable X
$\mathrm{F}(x)$, $\mathrm{G}(x)\ldots$	the value of the (cumulative) distribution function $\mathrm{P}(X \leqslant x)$ of the random variable X
$\mathrm{E}(X)$	expectation of the random variable X
$\mathrm{E}[\mathrm{g}(X)]$	expectation of $\mathrm{g}(X)$
$\mathrm{Var}(X)$	variance of the random variable X
$\mathrm{B}(n, p)$	binominal distribution, parameters n and p
$\mathrm{Po}(\mu)$	Poisson distribution, mean μ
$\mathrm{N}(\mu, \sigma^2)$	normal distribution, mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x}$	sample mean
s^2	unbiased estimate of population variance from a sample, $s^2 = \frac{1}{n-1}\sum(x-\bar{x})^2$
ϕ	probability density function of the standardised normal variable with distribution $\mathrm{N}(0, 1)$
Φ	corresponding cumulative distribution function
ρ	linear product-moment correlation coefficient for a population
r	linear product-moment correlation coefficient for a sample